

# Some Consequences of Dark Energy Density varying Exponentially with Scale Factor

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## Abstract

In this paper we have explored the consequences of a model of dark energy with its energy density varying exponentially with the scale factor. We first consider the model with  $\rho_\phi \propto e^{\kappa a}$ , where  $\kappa$  is a constant. This is a kind of generalisation of the cosmological constant model with  $\kappa = 0$ . We show that such an exponentially varying dark energy density with the scale factor naturally leads to an equivalent phantom field. We also consider a model with  $\rho_\phi \propto e^{\kappa/a}$  and we show that this also naturally leads to an equivalent phantom field.

# 1 Introduction

Observations on Supernova Type Ia, CMB and studies on large scale structure formation have strongly suggested that at the present epoch the universe is in a phase of accelerated expansion [1, 2]. Within the standard framework of General Relativity, this kind of expansion is not possible with the conventional kind of source, *i.e.*, which has an equation of state of the form  $p = w\rho$  with  $0 \leq w \leq 1$ . The present accelerated expansion of the universe seems to indicate that there exists some form of matter-energy with negative pressure which is causing this acceleration. (One could, of course, argue that Einstein's general relativity may not be the fully correct/complete theory on the cosmological scales [3]. But we will not consider this possibility in this paper.) The former possibility has motivated the exploration of alternative kinds of sources with  $p < -\frac{1}{3}\rho$  as well as a resurgence of investigations using cosmological constant ( $w = -1$ ). Such a matter energy component with equation of state  $p < -\frac{1}{3}\rho$  is generally known as 'Dark energy'. Several dark energy models have been considered in literature. The cosmological constant is just the simplest case with  $w = -1$ . Models with cosmological constant predict a transition to an accelerated expansion phase of the Universe.

Introducing the cosmological constant, however, leads to two serious problems. The first one is that the observations indicate that the value of  $\Lambda$  is many orders of magnitude smaller than what is expected from the energy scales involved in the Early Universe. This problem is known as the cosmological constant problem [4, 5, 6]. The second problem is that the observations indicate that at the present epoch the energy density of the  $\Lambda$  term, *i.e.*,  $\rho_\Lambda = \frac{\Lambda}{8\pi G}$  is of the same order in magnitude as the energy density of

matter  $\rho_m$  with  $\Lambda$  having begun to dominate in the recent past. This requires extreme fine tuning of the value of  $\Lambda$ . This problem is known as the cosmic coincidence problem. The vacuum energy density  $\rho_\Lambda$  does not change with the expansion of the Universe. Hence, there is a need to explore more general models of dark energy.

The other models of dark energy in which the density of dark energy,  $\rho_{de}$ , changes as the universe expands are based on Quintessence, Phantom field, Tachyon field etc. Quintessence models refer to those models in which dark energy is represented by a standard scalar field with a potential  $V(\phi)$  [7, 8, 9]. Specific forms of potential which satisfy certain conditions admit tracking solution which may possibly solve the cosmic coincidence problem [10, 11]. Quintessence models have an equation of state parameter  $w_{de}$  greater than minus one. Observational constraints on  $w_{de}$  do not exclude the possibility of  $w_{de}$  being less than minus one at the present epoch [12, 13]. This leads to the possibility that the present cosmic acceleration may be driven by a phantom field ( $w_{de} < -1$ ). Energy density of the phantom field increases as the universe expands. The phantom energy could be obtained from either a scalar field with negative kinetic energy [14, 15] term or by some form of non minimal coupling of the scalar field with gravity [16]. The phantom field described in this paper refer to that with a negative kinetic energy term. Some models of phantom energy with a constant  $w_{de}$  lead to a singularity after a finite time in the future [17, 18]. This is called the Big Rip. Certain other models with dynamical  $w_{de}$  do not lead to Big Rip [19]. There is yet another theoretical possibility that the present accelerated expansion might be due to tachyon fields [20, 21]. These were originally motivated from the

string theory. In these models  $w_{de} > -1$ .

In all these scalar field models the equation of state is given as  $p_\phi = w_\phi \rho_\phi$ . Dark energy with different equation of state has also been considered in literature, for example, Chaplygin gas with  $p = \frac{\alpha}{\rho}$  [23, 24] and models of dark energy with generalised equation of state  $p = \alpha(\rho - \rho_o)$  [22].

It has been argued that probing the dark energy via its density evolution rather than its equation of state has a number of advantages [25]. In this paper we consider a model of dark energy with its energy density varying exponentially with the scale factor. After discussing the basic formalism for a general scalar field in section 2, we will consider in section 3 the model with  $\rho_\phi \propto e^{\kappa a}$  where  $\kappa$  is a constant. This is a kind of generalisation of the cosmological constant model which corresponds to  $\kappa = 0$ . We will show that such a model naturally leads to an equivalent phantom field. This model, however, leads to Big Rip. This motivated us to consider another model with  $\rho_\phi \propto e^{\kappa/a}$  which is discussed in the section 4. This also naturally leads to an equivalent phantom field but it doesn't lead to Big Rip. It asymptotically behaves as a de Sitter Universe. The phase space analysis has been discussed in section 5

## 2 Energy-momentum Tensor and the Equations of Motion for a general scalar field

Let the Lagrangian density of the scalar field be given by  $\mathcal{L}_\phi$  and that for the rest of the matter by,  $\mathcal{L}_{source}$ . The complete Einstein-Hilbert action for

this system is then given by,

$$\mathcal{S} = \int \left[ \frac{-1}{16\pi G} R + \mathcal{L}_\phi + \mathcal{L}_{source} \right] \sqrt{-g} d^4x \quad (1)$$

We employ the metric with signature  $(+---)$ . The Lagrangian density  $\mathcal{L}_\phi$  for a normal scalar field is given by,

$$\mathcal{L}_\phi = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) \quad (2)$$

The kinetic energy term for this scalar field is positive. Another class of scalar fields have been considered in literature for which the kinetic energy term is a negative. These are called phantom fields and their Lagrangian density is given by,

$$\mathcal{L}_\phi = -\frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) \quad (3)$$

In order to incorporate both these kinds of scalar fields, we consider an action of the form,

$$\mathcal{S} = \int \left\{ \frac{-1}{16\pi G} R + \left[ \frac{\alpha}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) \right] + \mathcal{L}_{source} \right\} \sqrt{-g} d^4x \quad (4)$$

where  $\alpha$  takes the value  $+1$  for normal scalar field (quintessence) and  $-1$  for phantom field. The energy momentum tensor of the scalar field obtained from the action in equation (4) is given by:

$$T_{(\phi)}^{\mu\nu} = \alpha \partial^\mu \phi \partial^\nu \phi - g^{\mu\nu} \left[ \frac{\alpha}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) \right] \quad (5)$$

Consider the spatially flat FRW metric:

$$ds^2 = dt^2 - a(t)^2 [dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2] \quad (6)$$

We further assume that the scalar field is homogeneous. The spatial derivatives of such a field vanishes and the field depends only on time *i.e*

$\phi = \phi(t)$ . Using this form of  $\phi$  and the metric corresponding to the line element in equation (6), the energy momentum tensor in equation (5) assumes a diagonal form given by:

$$T_{(\phi)\nu}^{\mu} = \text{diag}(\rho_{\phi}, -p_{\phi}, -p_{\phi}, -p_{\phi}), \quad (7)$$

where,

$$\rho_{\phi} = \alpha \frac{\dot{\phi}^2}{2} + V(\phi) \quad (8)$$

and

$$p_{\phi} = \alpha \frac{\dot{\phi}^2}{2} - V(\phi) \quad (9)$$

The equation of state is given by  $p_{\phi} = w_{\phi} \rho_{\phi}$  where,

$$w_{\phi} = \frac{\alpha \frac{\dot{\phi}^2}{2} - V(\phi)}{\alpha \frac{\dot{\phi}^2}{2} + V(\phi)} \quad (10)$$

The dynamics of the Universe is determined by the Friedmann equations:

$$H^2 = \frac{8\pi G}{3} \rho \quad (11)$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} (\rho + 3p) \quad (12)$$

where  $\rho$  and  $p$  are the total energy density and the total pressure. ( We have assumed a  $k = 0$  universe). The field equation for  $\phi$  is obtained from the conservation equation  $T_{\phi}^{\mu\nu}{}_{;\nu} = 0$  and is given by:

$$\ddot{\phi} + 3H\dot{\phi} + \alpha^{-1} \frac{dV}{d\phi} = 0 \quad (13)$$

Let  $\rho = \rho_m + \rho_{\phi}$  (We have neglected the contribution from radiation since we are interested only in the late time evolution of the Universe). The scale

factor is normalised so as to make  $a(t_0) = 1$ . The density of matter is hence given by

$$\rho_m = \rho_{mo} a^{-3} \quad (14)$$

where  $\rho_{mo}$  is the matter density at the present epoch.

### 3 Model with $\rho_\phi \propto e^{\kappa a}$

As we stated in the introduction, our approach in this paper is to start with a form for the evolution of the energy density of the scalar field rather than the equation of state. This is the approach that has been adopted in [25]. The present paper is in this spirit. We consider a scalar field  $\phi$  in a potential  $V(\phi)$  whose energy density varies with scale factor as  $\rho_\phi \propto e^{\kappa a}$ . Since we have normalised the scale factor to unity today, we may write,

$$\rho_\phi = \rho_{\phi 0} e^{\kappa(a-1)} \quad (15)$$

where  $\rho_{\phi 0}$  is the dark energy density in the present epoch with the scale factor today normalised to unity. The continuity equation for the scalar field  $T_\phi^{\mu\nu}{}_{;\nu} = 0$  implies:

$$\frac{d\rho_\phi}{da} = -\frac{3}{a}(1 + w_\phi)\rho_\phi \quad (16)$$

where,  $w_\phi$  is the ratio of the pressure,  $p_\phi$ , to the energy density,  $\rho_\phi$ , of the scalar field. Using the form for  $\rho_\phi$  as given in equation (15), we get,

$$w_\phi = -1 - \frac{a\kappa}{3} \quad (17)$$

From equation (17) it follows that if  $\kappa < 0$  then at sufficiently large value of the scale factor  $a$ , the ratio of the pressure to density of the scalar field,

$w_\phi$  would become greater than one. This would violate causality. In order to ensure causality we need  $w_\phi < +1$  and in order to ensure this at all future epochs, we require  $\kappa$  to be greater than zero.

Using equations (8) and (10), we have

$$\alpha \dot{\phi}^2 = (1 + w_\phi) \rho_\phi \quad (18)$$

Substituting for  $w_\phi$  from equation(17), we get,

$$\alpha \dot{\phi}^2 = -\frac{a\kappa}{3} \rho_\phi \quad (19)$$

For  $\rho_\phi > 0$  and  $\kappa > 0$  (as required by the causality condition ),  $\alpha$  should be negative. Hence, the kinetic energy term must have a negative sign which fixes  $\alpha = -1$ . Hence such a form of  $\rho_\phi$  naturally leads to an equivalent phantom scalar field.

We now explore the nature of the potential  $V(\phi)$  of this phantom field which would lead to such a form of  $\rho_\phi$ . From equations (8), (9) and (10) (with  $\alpha = -1$ ) we have,

$$\rho_\phi = -\frac{\dot{\phi}^2}{2} + V(\phi) \quad (20)$$

$$p_\phi = -\frac{\dot{\phi}^2}{2} - V(\phi) \quad (21)$$

$$w_\phi = \frac{-\frac{\dot{\phi}^2}{2} - V(\phi)}{-\frac{\dot{\phi}^2}{2} + V(\phi)} \quad (22)$$

$$(23)$$

These equations imply,

$$\dot{\phi}^2 = -(1 + w_\phi) \rho_\phi \quad (24)$$

$$V(\phi) = \frac{(1 - w_\phi) \rho_\phi}{2} \quad (25)$$



The above equation (25) together with equations (15) and (17) gives the potential  $V(\phi)$  as a function of the scale factor  $a$ . The derivative of  $\phi$  with respect to scale factor  $a$  is given by:

$$\frac{d\phi}{da} = \frac{\dot{\phi}}{\dot{a}} = \frac{\dot{\phi}}{aH} \quad (26)$$

where  $H$  is given by the Friedmann equation (11):

$$H^2 = \frac{8\pi G}{3}(\rho_{m0}a^{-3} + \rho_{\phi0}e^{\kappa(a-1)}) \quad (27)$$

Using equations (15), (17), (24), (25), (26) and (27) one obtains,

$$\frac{d}{da}\left(\frac{\phi}{M_p}\right) = \frac{a(\kappa\Omega_\phi)^{1/2}}{(\Omega_\phi a^3 + \Omega_m e^{-\kappa(a-1)})^{1/2}} \quad (28)$$

$$V(a) = 3\Omega_\phi H_0^2 M_p^2 \left(1 + \frac{a\kappa}{6}\right) e^{\kappa(a-1)}. \quad (29)$$

Here,

$$\Omega_\phi = \frac{\rho_{\phi0}}{3H_0^2 M_p^2} \quad (30)$$

and

$$\Omega_m = \frac{\rho_{m0}}{3H_0^2 M_p^2} \quad (31)$$

And

$$M_p^2 = \frac{1}{8\pi G} \quad (32)$$

which is the reduced Planck mass in units of  $\hbar = c = 1$ .  $H_0$  is the Hubble parameter. Using equations (28) & (29) one can plot potential  $V(\phi)$  as a function of  $\phi$ . This plot is shown in figure (1).

In this plot we have chosen initial condition such that  $\phi = 0$  at  $a = 10^{-5}$ . The field  $\phi$  grows from  $\phi = 0$  to a value  $\phi = 0.55M_p$  at the present epoch ( $a = 1$ ).

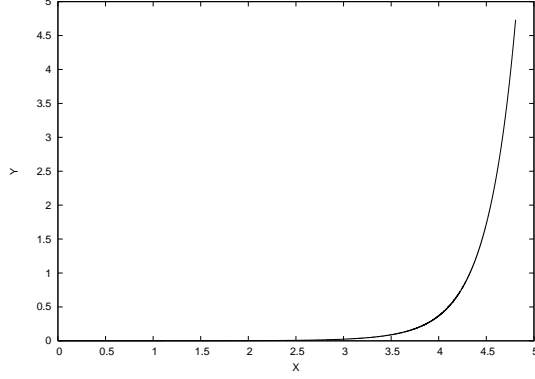


Figure 1: This figure shows the potential  $V(\phi)$  for the model  $\rho_\phi \propto e^{\kappa a}$ . In this figure  $X = \frac{\phi}{M_{pl}}$  and  $Y = \frac{V(\phi)}{H_0^2 M_{pl}^2} 10^{-4}$ . For this plot we have taken  $\kappa = 1$ ,  $\Omega_\phi = 0.73$  and  $\Omega_m = 0.27$ .

At low value of the field *i.e.* for  $\phi \ll M_p$  which occurs at  $a \ll 1$ , the differential equation (28) can be approximated as

$$\frac{d}{da} \left( \frac{\phi}{M_p} \right) \simeq a \sqrt{\frac{\kappa \Omega_\phi}{\Omega_m e^\kappa}}, \quad (33)$$

which on integration gives:

$$a = \sqrt{\frac{\alpha}{M_p}} (\phi - \phi_0) \quad (34)$$

Where  $\phi_0$  is the constant of integration and

$$\alpha = 2 \sqrt{\frac{\Omega_m e^\kappa}{\kappa \Omega_\phi}} \quad (35)$$

On substituting equation (34) in the equation (29), we obtain an approximate functional form of  $V(\phi)$  for small value of the field ( $\phi \ll M_p$ ). This is given as:

$$V(\phi) = V_0 \left( 1 + \frac{\kappa}{6} \sqrt{\frac{\alpha}{M_p}} (\phi - \phi_0) \right) \exp \left( \kappa \sqrt{\frac{\alpha}{M_p}} (\phi - \phi_0) \right), \quad (36)$$

where

$$V_0 = 3\Omega_\phi H_0^2 M_p^2 e^{-\kappa} \quad (37)$$

Similarly for large value of the field, *i.e.* for  $\phi > M_p$ , which occurs at  $a > 1$  (and hence at a future epoch), the differential equation (28) can be approximated as:

$$\frac{d}{da}\left(\frac{\phi}{M_p}\right) \simeq \sqrt{\frac{\kappa}{a}}, \quad (38)$$

which on integration gives:

$$a = \frac{(\phi - \phi_0)^2}{4\kappa M_p^2}. \quad (39)$$

Here  $\phi_0$  is the constant of integration. Substituting equation (39) in the equation (29), we obtain an approximate functional form of the potential  $V(\phi)$  at large value of the field  $\phi$ . This is given as:

$$V(\phi) = V_0 \left(1 + \frac{(\phi - \phi_0)^2}{24M_p^2}\right) \exp\left(\frac{(\phi - \phi_0)^2}{4M_p^2}\right) \quad (40)$$

We will consider this form of potential given by equation (40) in section 5 to study the phase space analysis.

The value of the constant  $\kappa$  can be determined from the epoch of equality of matter and dark energy density. Let  $z_{m\phi}$  be the red shift of the epoch of matter-dark energy equality. Equating the expression for  $\rho_\phi$  and  $\rho_m$  from equations (15) and (14) we get,

$$\kappa = -\left(1 + \frac{1}{z_{m\phi}}\right) \ln\left(\frac{\Omega_m}{\Omega_\phi}(1 + z_{m\phi})^3\right) \quad (41)$$

As  $\kappa > 0$  equation(41) implies,

$$z_{m\phi} < \left(\frac{\Omega_\phi}{\Omega_m}\right)^{1/3} - 1 \quad (42)$$

As  $\Omega_\phi = 0.73$  and  $\Omega_m = 0.27$  (as indicated by the observations by WMAP [26]) we have  $z_{m\phi} < 0.393$ . This model (with  $\rho_\phi \propto e^{\kappa a}$ ), thus, constrains the value of  $z_{m\phi}$ . We can see from the expression of  $w_\phi(a)$  given by equation(17) that  $a \rightarrow \infty \Rightarrow w_\phi(a) \rightarrow -\infty$ . From the expression of  $\rho_\phi$  (15) and the equation of state we see that  $\rho_\phi \rightarrow \infty$  and  $p_\phi \rightarrow -\infty$  as  $a \rightarrow \infty$ . This would then mean that the trace of the energy momentum tensor  $T = T^\mu_\mu = \rho_\phi - 3p_\phi$  (and hence the Ricci scalar  $R \propto T$ ) would blow up as  $a \rightarrow \infty$ . It is of interest to ascertain the epoch when the scale factor tends to infinity. In particular, we would like to know whether this happens at a finite time or asymptotically in the infinite future.

The epoch,  $t_\infty$ , when the scale factor becomes infinitely large can be calculated from the Friedman equation (27) to be,

$$t_\infty = t_0 + \frac{1}{H_0} \int_1^\infty \frac{da}{a(\sqrt{\Omega_m a^{-3} + \Omega_\phi e^{\kappa(a-1)}})} \quad (43)$$

where  $t_\infty$  is the epoch when  $a \rightarrow \infty$ . As the scale factor becomes infinitely large, the first term under the square root term becomes subdominant as compared to the second and so the integral remains finite. Thus this model exhibits a singularity at a finite future epoch. This is popularly known as Big Rip and our model  $\rho_\phi \propto e^{\kappa a}$  exhibits that feature.

## 4 Model with $\rho_\phi \propto e^{\kappa/a}$

Let  $\rho = \rho_m + \rho_\phi$  where  $\rho_\phi \propto e^{\kappa/a}$ . Denoting by  $\rho_{\phi 0}$  the density of the scalar field in the present epoch corresponding to  $a = 1$ , we have:

$$\rho_\phi = \rho_{\phi 0} e^{\kappa \frac{1-a}{a}} \quad (44)$$

We again consider the action of the form (4). The continuity equation  $T_{\phi}^{\mu\nu}{}_{;\nu} = 0$  would imply that:

$$w_{\phi} = -1 + \frac{\kappa}{3a} \quad (45)$$

If  $\kappa > 0$  then at sufficiently low value of the scale factor  $a$ ,  $w_{\phi}$  would become greater than one. This would violate causality. So for  $w_{\phi}$  to be less than one requires  $\kappa$  to be less than zero. Hence in this model causality requires that  $\kappa < 0$ . With these constraints, we investigate the sign of the kinetic energy term of the scalar field that would lead to  $\rho_{\phi}$  of the form (44) and also calculate the form of the potential  $V(\phi)$  of the field  $\phi$ . Using equations (8), (9), (10), (44) & (45) we obtain,

$$\alpha\dot{\phi}^2 = \frac{\kappa}{3a}\rho_{\phi} \quad (46)$$

For  $\rho_{\phi} > 0$  and  $\kappa < 0$  (we have shown that for this model causality requires  $\kappa < 0$ ), the left hand side of the equation is negative. So in this case too, the kinetic energy term must be negative. Hence such a form of  $\rho_{\phi}$  also naturally leads to an equivalent phantom scalar field. We now obtain the form of the potential  $V(\phi)$  of the phantom field which would lead to such a form of  $\rho_{\phi}$ . For  $\rho_{\phi}$  given by equation (44), the Friedmann equation (27) becomes:

$$H^2 = \frac{8\pi G}{3}(\rho_{m0}a^{-3} + \rho_{\phi0}e^{\kappa\frac{1-a}{a}}) \quad (47)$$

Using equations (44), (45), (24), (25), (26) and (47) we obtain,

$$\frac{d}{da}\left(\frac{\phi}{M_p}\right) = \sqrt{\frac{-\kappa\Omega_{\phi}}{\Omega_{\phi}a^3 + \Omega_m e^{-\kappa(\frac{1}{a}-1)}}} \quad (48)$$

$$\frac{V(a)}{H_0^2 M_p^2} = 3\Omega_{\phi}\left(1 - \frac{\kappa}{6a}\right)e^{\kappa(\frac{1}{a}-1)} \quad (49)$$

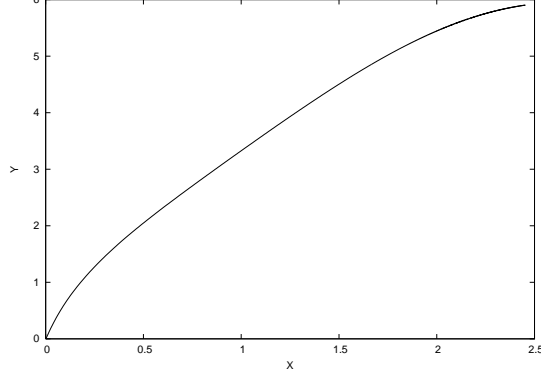


Figure 2: This figure shows the potential  $V(\phi)$  for the model  $\rho_\phi \propto e^{k/a}$ . In this figure  $X = \frac{\phi}{M_{pl}}$  and  $Y = \frac{V(\phi)}{H_0^2 M_{pl}^2}$ . For this plot we have taken  $\kappa = -1$ ,  $\Omega_\phi = 0.73$  and  $\Omega_m = 0.27$ .

Using equations (48) & (49) one can plot potential  $V(\phi)$  as a function of  $\phi$ . This plot is shown in figure (2). In this plot also we have chosen an initial condition such that  $\phi = 0$  at  $a = 10^{-5}$ . This gives a value of the field  $\phi = 0.7M_p$  at the present epoch ( $a = 1$ ). The field  $\phi$  grows from  $\phi = 0$  to a value  $\phi \simeq 2.45M_p$ . As done in the previous section we obtain the limiting form of the potential  $V(\phi)$  from equations (48) and (49).

When  $\phi \ll M_p$  which occurs at  $a \ll 1$ , the potential  $V(\phi)$  is nearly flat. When  $\phi > M_p$  and  $a \gg 1$ , the equation (48) can be approximated as:

$$\frac{d}{da}\left(\frac{\phi}{M_p}\right) \simeq \sqrt{\frac{-\kappa}{a^3}}, \quad (50)$$

which on integration gives:

$$a = \frac{-4\kappa M_p^2}{(\phi - \phi_0)^2}. \quad (51)$$

Here  $\phi_0$  is the constant of integration. Substituting this equation in equation (49), we obtain a limiting form of the potential at large value of the scale

factor and for  $\phi > M_p$ . This is given as:

$$V(\phi) = V_0(1 + \frac{(\phi - \phi_0)^2}{24M_p^2})\exp(-\frac{(\phi - \phi_0)^2}{4M_p^2}) \quad (52)$$

where

$$V_0 = 3\Omega_\phi H_0^2 M_p^2 e^{-\kappa} \quad (53)$$

It is interesting to note that this limiting form of the potential given by equation (52) is very similar to that obtained in the previous section ( equation (40)) with the only difference in the sign in the exponential part. We will also consider this form of the potential given by equation (52) in section 5 to study the phase space analysis.

In this model also we can obtain the value of the constant  $\kappa$  from the red shift  $z_{m\phi}$  of the epoch of matter-dark energy equality. Equating the expressions for  $\rho_\phi$  and  $\rho_m$  we obtain:

$$\kappa = \frac{1}{z_{m\phi}} \ln\left(\frac{\Omega_m}{\Omega_\phi}(1 + z_{m\phi})^3\right) \quad (54)$$

As  $\kappa < 0$ , the above equation (54) would imply that:

$$z_{m\phi} < \left(\frac{\Omega_\phi}{\Omega_m}\right)^{1/3} - 1 \quad (55)$$

This is same as equation (42). Hence we can see that both this model and the model considered in the previous section puts same constraints on  $z_{m\phi}$ . Using equations (44) & (45) we can see that this model ( $\rho_\phi \propto e^{\kappa/a}$ ) asymptotically behaves as a De Sitter's Universe. So this model does not lead to Big Rip. This is unlike the previous model which exhibits a Big Rip.

## 5 Phase space analysis

As the Universe expands, the relative contribution of pressure-less matter keeps decreasing and as the scale factor becomes very large, the matter content can be neglected. Let us consider the model with  $\rho_\phi \propto e^{\kappa a}$  and analyse the situation in this limit of large  $a$ . In this limit, we neglect the energy density contribution of matter and only consider that due to scalar field and integrate the equation of motion for the scalar field to obtain.

$$\phi(a) = 2\sqrt{\frac{\kappa a}{8\pi G}} + \phi_0 \quad (56)$$

where  $\phi_0$  is the constant of integration. Using this equation and the equation (29) we obtain the following equation:

$$V(\phi) = V_0(1 + \frac{\phi^2}{24M_p^2})\exp(\frac{\phi^2}{4M_p^2}) \quad (57)$$

where

$$V_0 = 3H_0^2 M_p^2 e^{-\kappa} \quad (58)$$

(Equation (57) is the same as the equation (40) but in the equation (57) we have taken  $\phi_0$  to be zero ). From equation (17) the redshift dependence of  $w_\phi(a)$  is of the form,  $w_\phi = -1 + ma$  where  $m$  is a constant and  $a$  is the scale factor, together with the continuity equation,

$$\frac{d}{da}(\rho a^3) + 3pa^2 = 0 \quad (59)$$

this form of  $w_\phi(a)$  imposes,  $\rho_\phi = Ae^{-3ma}$ . So in order to investigate whether different initial condition leads to  $\rho_\phi$  of the form  $\rho_\phi \propto e^{\kappa a}$ , we plot  $w_\phi$  as a function of the scale factor for different initial conditions. This plot is shown in fig(3). From this we can conclude that for different initial condition  $w(a)$  is



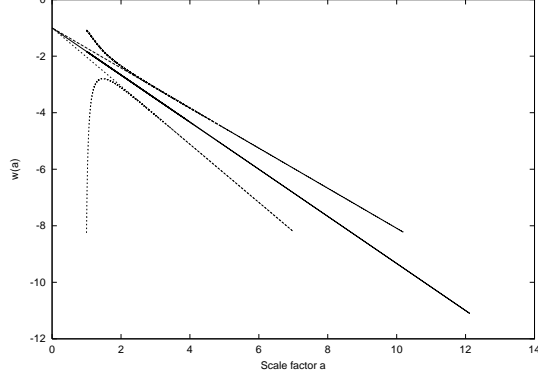


Figure 3: Plot of equation of state parameter  $w(a)$  verses scale factor for different initial conditions for the potential given by equation (57). Here  $\rho_m = 0$  and  $\kappa = 2.5$ .

of the form  $w(a) = -1 + ma$  asymptotically with the constant  $m$  depending on the initial condition. Hence, asymptotically  $\rho_\phi$  will be of the form  $\rho_\phi = Ae^{\kappa a}$  for different initial conditions but both the constants  $\kappa$  and  $A$  depends on the initial condition. Phase portrait (for the case when the potential is given by equation (57)) is shown in fig(4).

Similarly for the model  $\rho_\phi \propto e^{\kappa/a}$ , in the limit of large  $a$ , we get,

$$V(\phi) = V_0 \left(1 + \frac{\phi^2}{24M_p^2}\right) \exp\left(-\frac{\phi^2}{4M_p^2}\right) \quad (60)$$

where

$$V_0 = 3H_0^2 M_p^2 e^{-\kappa} \quad (61)$$

(Equation (60) is the same as the equation (52) but in the equation (60) we have taken  $\phi_0$  to be zero ). Here we plot  $w(a)$  verses  $1/a$  for different initial conditions. This plot is shown in the fig(5). Here also we can conclude that

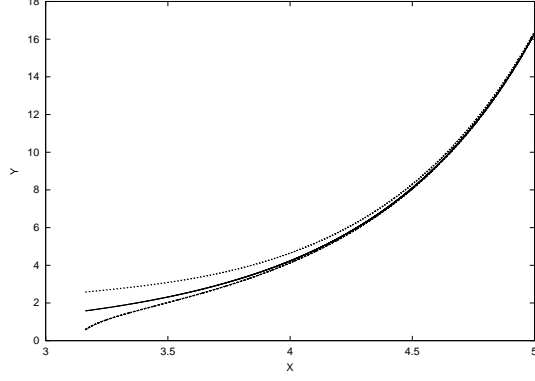


Figure 4: Phase portrait in model with potential given by equation (57). Here  $\rho_m = 0$  and  $\kappa = 2.5$ . The initial condition chosen is the same as in fig(3). Here  $X = \phi/M_p$  and  $Y = \dot{\phi}/H_0 M_p$

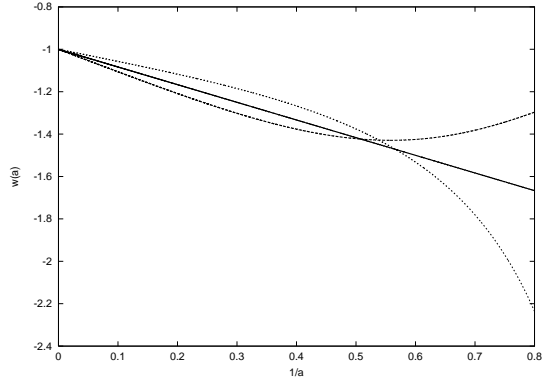


Figure 5: Plot of equation of state parameter  $w(a)$  verses inverse of scale factor for different initial conditions for the potential given by equation (60). Here  $\rho_m = 0$  and  $\kappa = -2.5$ . Here  $1/a \rightarrow 0 \Rightarrow$  asymptotically into the future.

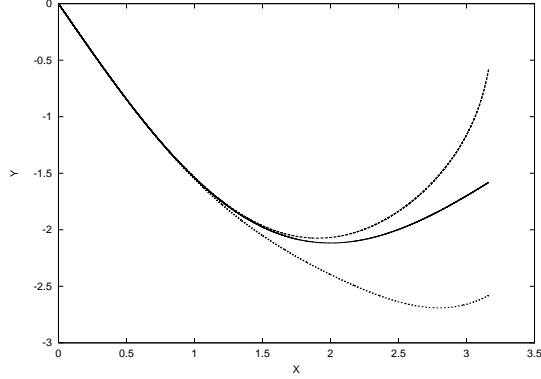


Figure 6: Phase portrait in model with potential given by equation (60). Here  $\rho_m = 0$  and  $\kappa = -2.5$ . The initial condition chosen is the same as in fig(5). Here  $X = \phi/M_p$  and  $Y = \dot{\phi}/H_0 M_p$

for different initial conditions  $w(a)$  is of the form  $w(a) = -1 + \frac{m}{a}$  asymptotically with the constant  $m$  depending on the initial condition. We conclude that asymptotically  $\rho_\phi$  will be of the form  $\rho_\phi = Ae^{\kappa/a}$  for different initial conditions but both the constants  $\kappa$  and  $A$  depends on the initial condition. Phase portrait (for the case when the potential is given by equation (60)) is shown in fig(6).

## 6 Conclusions

In this paper we have shown that the dark energy density varying exponentially as  $\rho_\phi \propto e^{\kappa a}$  and  $\rho_\phi \propto e^{\kappa/a}$  naturally lead to an equivalent phantom field. We have shown in section 5 that in the limit of large  $a$  phantom potential of the form given by equation (57) leads to  $\rho_\phi \propto e^{\kappa a}$  for a particular initial condition on  $\phi$  and  $\dot{\phi}$ . For other nearby initial conditions  $\rho_\phi$  tends to this

form asymptotically and the constant  $\kappa$  depends on the initial condition ( $\phi$  and  $\dot{\phi}$ ). This model leads to Big Rip. Similarly in the large  $a$  limit, phantom potential of the form given by equation (60) leads to  $\rho_\phi \propto e^{\kappa/a}$  for certain initial conditions. For other nearby initial conditions  $\rho_\phi$  approaches the form  $\rho_\phi \propto e^{\kappa/a}$  asymptotically and the constant  $\kappa$  depends on the initial conditions ( $\phi$  and  $\dot{\phi}$ ). It does not leads to Big Rip but rather it asymptotically leads to De Sitter Universe.

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## References

- [1] A.G.Riess, *et al.*, *Astron.J.* **116**,1009 (1998).
- [2] S Perlmutter, *et al.*, *Ap.J.* **517**,565 (1999).
- [3] A.Lue, R.Soccimarro and G.Starkman,*Phys. Rev.* **D69**, 044005.(2004).
- [4] S.Weinberg, *Rev. Mod. Phys.* **61**, 1(1989).
- [5] T.Padmanabhan, *Phys. Report*, **380**, 235(2003).
- [6] P.J.E.Peebles and B.Ratra, *Rev. Mod. Phys.* **75**,559(2003).
- [7] P.G.Ferreira and M Joyce,*Phys. Rev.* **D58**,023503(1998).

- [8] M.S.Turner, *Phys. Rev.* **D28**,1243(1983).
- [9] B.Ratra and P.J.E.Peebles, *Phys. Rev.* **D37**,3406(1988).
- [10] P.J.E.Peebles and B.Ratra, *Ap.J. Lett.* **325**,L 17 (1988).
- [11] P.J.Steinhardt, L.Wang and I Zlatev, *Phys. Rev.* **D59**,123504(1999).
- [12] A.G.Riess, *et al.*,*Ap.J.* **607**,665(2004).
- [13] R.A.Knop, *et al.*, *Ap.J.*, **598**,102(2003).
- [14] S.M.Carroll, M.Hoffman and M.Trodden *Phys. Rev.* **D68**,  
023509(2003).
- [15] M.Kaplinghat and S.Bridle, astro-ph/0312430
- [16] F.C. Carvalho and A.Saa, *Phys. Rev.* **D70**,087302(2004).
- [17] J.D.Barrow, *Class. Quantum Grav.* **21**,L79 (2004).
- [18] R.R.Caldwell, *Phys. Lett.* **B545**,23(2002).
- [19] M.Sami and A.Toporensky, *Mod.Phys. Lett.* **A19**,1509(2004).
- [20] T.Padmanabhan, *Phys. Rev.* **D66**,021301(2002).
- [21] J.S.Bagla, H.K.Jassal and T.Padmanabhan, *Phys. Rev.* **D67**,  
063504(2003).
- [22] E.Babichev, V.Dokuchaev and Yu.Eroshenko,*Class. Quantum Grav.* **22**,  
143(2005).

- [23] A.Kamenshchik, U.Moschella and V.Pasquier, *Phys. Lett.* **B511**, 265(2001).
- [24] N.Bilic, G.B.Tupper, R.D.Viollier, *Phys. Lett.* **B535**, 17(2002).
- [25] Y.Wang and K.Freese, astro-ph/0402208
- [26] C. L. Bennett *et al.* *Ap.J.Sup.* **148**, 1 (2003).